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ENERGY CHANGES IN BAROCLINIC MODELS

KELVIN WAYNE HUEHN

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ENERGY CHANGES IN BAROCLINIC MODELS

by

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Submitted in partial fulfillment
for the degree of

MASTER OF SCIENCE IN METEOROLOGY

from the

UNITED STATES NAVAL POSTGRADUATE SCHOOL
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ABSTRACT

The effects of heat transfer on the kinetic energy tendencies of a perturbation superimposed on a basic zonal current are investigated. The rate at which kinetic energy is gained in both the adiabatic and diabatic models increases with time for medium wave lengths, but the rate of increase is less for the diabatic case. At the longer wave lengths the energy change for the diabatic model shows a decrease with time becoming negative toward the end of the time interval investigated. The adiabatic model, in contrast, has zero energy tendency at long wave lengths. The rate of kinetic energy change increases as larger values of thermal wind are selected, and for the adiabatic model the wave length range for which energy changes occur also expands with increasing thermal wind. The energy gain initially is a maximum when the thermal wave lags the pressure wave by a phase angle of 90 degrees and the maximum tendency to lose energy occurs when the lag is 270 degrees.

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TABLE OF SYMBOLS

J = the relative vorticity

$\mathbf{V}$ = the wind velocity vector

f_0 = the average value of the Coriolis parameter and equal to the the value at latitude 45N

∇ = the gradient operator in pressure coordinates

g = the acceleration of gravity

z = the height of the pressure surface

T = the temperature

ω = the total derivative of pressure with respect to time (vertical velocity)

S = the static stability parameter $\left(\frac{\partial T}{\partial p} - \frac{\alpha}{c_p}\right)$

β = the Rossby parameter

R = the gas constant for dry air

J = the Jacobian operator

μ^2 = a measure of static stability $\left(\frac{-f_0^2}{pSR}\right)$

c_p = the specific heat for constant pressure

ψ = the stream function $\left(\frac{1}{2}\frac{g}{f_0}\left[z_1+z_3\right]\right)$

π = the thickness function $\left(\frac{g}{f_0}\left[z_1-z_3\right]\right)$ until specified as π = 3.14159

$\frac{dq}{dt}$ = the heat gain or loss

p = the atmospheric pressure

k = the wave number $\left(\frac{2\pi}{L}\right)$

L = the wave length

c = the phase speed of the waves

t = time

$\bar{u}$ = the basic zonal wind

u^* = the basic thermal wind

TABLE OF SYMBOLS
(Continued)

α = the relative phase angle between temperature and pressure wave

i = the square root of -1

I. Introduction

To date simple baroclinic models have been unable to improve significantly on the results obtained by one parameter models in the prognosis of mid-tropospheric circulation. Recent investigations have shown that the incorporation of previously neglected terms, which were felt to be negligible for short range forecasting, can significantly alter the characteristics of synoptic waves produced by baroclinic forecasting models, thus bringing these waves into closer proximity with snyoptic waves observed in nature.

In one of these investigations, Haltiner [2] considered the influence of sensible heat exchange on the properties of harmonic perturbations superimposed on a zonal current. The study revealed that this type of heating reduces the instability of short and medium waves, but that the instability of long waves is increased.

The purpose of this work is to compare the kinetic energy changes averaged over the interval of one wave length in a baroclinic model with heat transfer present to one when heating is absent. In particular, the manner in which heating alters the instability of long waves will be examined. A simple two-level quasi-geostrophic model is constructed and an equation expressing the time rate of change of kinetic energy in a harmonic perturbation superimposed on a zonal current is derived. The kinetic energy is computed numerically for the range of wave lengths corresponding to synoptically observed waves. At each wave length computations are made for varying values of thermal wind and phase lag between the pressure wave and the temperature wave. The change of kinetic energy with time is evaluated in time steps of three hours over a thirty hour period and the value of the tendency for the heating and the non-

heating model are compared.

In addition, computations are made of the values of C_i/L, which is proportional to the growth rate of the wave, and the values of phase speed to verify that the results obtained from the simple model used are in qualitative agreement with those obtained by Haltiner.

Interest throughout the study is confined to unstable waves only and solutions which lead to neutral waves are eliminated.

II. Mathematical Development

The atmosphere is represented by a four-layer baroclinic model using the 750 and 250 millibar levels as data levels and centering attention at 500 millibars. Since the effects of latitude variation are well known, latitude is held fixed at 45 N. The vertical velocity is assumed to be zero at both the top and the bottom of the model atmosphere, the effects of friction at the earth's surface are neglected, and the static stability of the atmosphere is taken to be constant.

The vorticity equation is applied at the levels corresponding to 250 and 750 millibars in the following form:

$$\frac{\partial J}{\partial t} + \mathbf{V} \cdot \nabla (J + f) = f_0 \nabla \cdot \mathbf{V} \qquad (1)$$

The continuity equation is used to substitute for the divergence in the above equation and then the derivative of the vertical velocity with respect to pressure is approximated by a finite difference ratio. Geostrophic approximations are substituted for the vorticity and wind. The sum and difference of this equation at the data levels are computed with the following results:

$$\frac{g}{f_0}\left[\nabla^2\left(\frac{\partial z_1}{\partial t} + \frac{\partial z_3}{\partial t}\right) + J\left(z_1, \frac{g}{f_0}\nabla^2 z_1 + f\right) + J\left(z_3, \frac{g}{f_0}\nabla^2 z_3 + f\right)\right] = 0 \qquad (2)$$

$$\frac{g}{f_0}\left[\nabla^2\left(\frac{\partial z_1}{\partial t} - \frac{\partial z_3}{\partial t}\right) + J\left(z_1, \frac{g}{f_0}\nabla^2 z_1 + f\right) - J\left(z_3, \frac{g}{f_0}\nabla^2 z_3 + f\right)\right] = 0. \qquad (3)$$

At the middle level (500 mb) the First Law of Thermodynamics is applied in the following form:

$$\frac{\partial T_2}{\partial t} + \mathbf{V} \cdot \nabla T + S_2 \omega_2 = \frac{1}{c_p}\frac{dq_2}{dt} \qquad (4)$$

where

$$S_2 = \left(\frac{\partial T}{\partial p} - \frac{\alpha}{c_p}\right)_2 .$$

The temperature at this level is approximated using the equation of state and the hydrostatic equation to give

$$T_2 = \frac{g}{R}\left(z_1 - z_3\right).$$

This approximation is used in (4) and used to replace ω in (3). With the following substitutions:

$$\psi = \frac{1}{2}\frac{g}{f_0}\left(z_1 + z_3\right), \quad \pi = \frac{g}{f_0}\left(z_1 - z_3\right), \quad \mu^2 = \frac{-4f_0^2}{p_0 R S_2}$$

our system of equations becomes

$$\nabla^2\left(\frac{\partial \psi}{\partial t}\right) + J\left(\psi, \nabla^2\psi\right) + \frac{1}{4}J\left(\pi, \nabla^2\pi\right) + \beta\frac{\partial \psi}{\partial x} = 0 \tag{5}$$

$$\nabla^2\left(\frac{\partial \pi}{\partial t}\right) - \mu^2\left(\frac{\partial \pi}{\partial t}\right) + J\left(\psi, \nabla^2\pi\right) + J\left(\pi, \nabla^2\psi\right) + \beta\left(\frac{\partial \pi}{\partial t}\right) - \mu^2 J\left(\psi, \pi\right) + \mu^2\frac{R}{c_p f_0}\frac{dq_2}{dt} = 0. \tag{6}$$

Let the basic flow and the surface be considered to be in thermal equilibrium. Similar to Dòòs [1], the exchange of sensible heat is then assumed to be proportional to the difference between the surface perturbation air temperature (T_a) and the temperature of the ground (T_g) and a decreasing power of pressure in the following manner:

$$\frac{dq_2}{dt} = G\left(T_g - T_a\right)\left(\frac{p}{p_s}\right)^n \tag{7}$$

where G is a proportionality constant.

Since ridges in the thermal wave will correspond to heat sinks for the air, and troughs will be heat sources, let the transfer of heat be proportional and opposite in sign to the perturbation thickness. There-

fore

$$K\pi' = -\frac{R}{c_p \beta_0}\frac{dq_2}{dt} \tag{8}$$

where K is a proportionality constant with units sec^{-1}.

To linearize the system (5) and (6), assume that the solutions are of the form

$$\psi = \psi_0 e^{ik(x-ct)} \qquad \pi = \pi_0 e^{ik(x-ct)}$$

and that

$$\psi = \bar{\psi}(y) + \psi'(x,t) \qquad \pi = \bar{\pi}(y) + \pi'(x,t)$$

Dropping terms involving products of perturbation quantities leads to a homogeneous linear system MN=0, where M is the matrix

$$\begin{pmatrix} ik(c-\bar{u}) + ik\beta & -ik^3\frac{u^*}{4} \\ iku^*(\mu^2-k^2) & ik(c-\bar{u})(\mu^2+k^2) + ik\beta - \mu^2 K \end{pmatrix} \tag{9}$$

and N is a column matrix of ψ_0 and π_0. In order to have non-trivial solutions to this system of equations the determinant of the coefficient matrix M must vanish. Setting this determinant equal to zero yields the frequency equation

$$c = \bar{u} - \frac{1}{2}\left(\frac{\beta}{k^2} + \frac{\beta}{\mu^2+k^2} + \frac{i\mu^2 K}{k(\mu^2+k^2)}\right) \pm \left(\frac{1}{4}\left[\frac{u^{*2}(\beta^2 - 2i\beta kK - k^2K^2)}{k^4(\mu^2+k^2)^2}\right] - \frac{u^{*2}}{4}\frac{(\mu^2-k^2)}{\mu^2+k^2}\right)^{1/2} \tag{10}$$

where

$$\bar{u} = -\frac{\partial\bar{\psi}}{\partial y} \qquad u^* = -\frac{\partial\bar{\pi}}{\partial y} \qquad \beta = \frac{\partial f}{\partial y}.$$

It can be seen from this equation that the presence of heating insures that the phase velocity will be a complex number regardless of

the wave length. If the heating is taken to be zero, it can be shown that the phase velocity will be complex only for a certain range of wave lengths. The width of this range will depend on the strength of the thermal wind and the value of the static stability.

Using the primary roots of the complex number under the radical sign, the phase velocity has two solutions: C_+ when using the positive sign in front of the radical, and C_- when using the negative sign. The complete solutions for ψ and π are therefore given by

$$\psi = \psi_0^+ e^{ik(x-c_+t)} + \psi_0^- e^{ik(x-c_-t)} \tag{11}$$

and

$$\pi = \pi_0^+ e^{ik(x-c_+t)} + \pi_0^- e^{ik(x-c_-t)} \tag{12}$$

where the primes are omitted and henceforth ψ and π will be perturbation quantities unless otherwise specified.

Next take as initial conditions at time t=0

$$\psi = Ae^{ikx} \qquad \pi = Be^{i(kx+\alpha)}$$

where A and B are real numbers and α is a relative phase difference which is positive when the thermal field lags the stream field. From (11) and (12) it follows immediately that at t=0

$$\psi_0^+ + \psi_0^- = A \qquad \pi_0^+ + \pi_0^- = Be^{i\alpha} \tag{13}$$

Along with (13), use of the linearized form of (5), once for C_+ and once for C_-, yields four equations in ψ_0^+, ψ_0^-, π_0^+, and π_0^-. Letting C_R equal the Rossby wave speed, the following solutions result:

$$\psi_0^+ = A - \frac{\left[A(c_+ - c_R) - Be^{i\alpha}\frac{u^*}{4}\right]}{(c_+ - c_-)} \tag{14a}$$

$$\psi_0^- = \frac{\left[A(c_+ - c_R) - Be^{i\alpha}\frac{u^*}{4}\right]}{(c_+ - c_-)} \tag{14b}$$

$$\pi_0^+ = Be^{i\alpha} - \frac{\left[A(c_+ - c_R) - Be^{i\alpha}\frac{u^*}{4}\right]\left[c_- - c_R\right]}{\frac{u^*}{4}(c_+ - c_-)} \tag{14c}$$

$$\pi_0^- = \frac{\left[A(c_+ - c_R) - Be^{i\alpha}\frac{u^*}{4}\right]\left[c_- - c_R\right]}{\frac{u^*}{4}(c_+ - c_-)} \tag{14d}$$

The above solutions for ψ_0^+, ψ_0^-, π_0^+ and π_0^- are complex functions. Therefore, they can be separated into their real and imaginary parts

$$\psi_0^+ = \psi_{0R}^+ + i\,\psi_{0I}^+$$

and similarly for ψ_0^-, π_0^+ and π_0^-.

As mentioned previously, the phase velocity can be a complex number depending on whether heating is present and the values of the thermal wind and static stability. The phase velocity can then be represented by

$$c_+ = a_1 + ib + (a_2 + id) \qquad c_- = a_1 + ib - (a_2 + id) \tag{15}$$

where

$$a_1 = \bar{u} - \frac{1}{2}\left(\frac{\beta}{k^2} + \frac{\beta}{\mu^2 + k^2}\right) \qquad ib = -\frac{i\mu^2 K}{2k(\mu^2 + k^2)}$$

and

$$a_2 + id = \left(\frac{1}{4}\left[\frac{\mu^4(\beta^2 - 2i\beta k K - k^2K^2)}{k^4(\mu^2 + k^2)^2}\right] - \frac{u^{*2}}{4}\frac{\mu^2 - k^2}{(\mu^2 + k^2)}\right)^{1/2} \tag{16}$$

The kinetic energy of the perturbation may be expressed in terms of the square of the velocity of the perturbation wind. Since ψ is independent of y and considering a unit mass, the kinetic energy per unit mass is

$$KE = \frac{1}{2} V^2 = \frac{1}{2}\left(\frac{\partial \psi}{\partial x}\right)^2$$

A solution for V is obtained by substituting into (11) the solutions for C_+ and C_- as given in (15) and then differentiating (11) with respect to X. The real part of V is squared, multiplied by one half (1/2) and integrated over an interval of one wave length to give the average kinetic energy of the perturbation over one wave length as follows:

$$\begin{aligned}\frac{1}{2}\int_0^L V^2 dx = {} & \frac{\pi k}{2}\left[e^{2(b+d)kt}\left(\psi_{OR}^{+2} + \psi_{OI}^{+2}\right) + e^{2(b-d)kt}\left(\psi_{OR}^{-2} + \psi_{OI}^{-2}\right)\right] \\ & + \pi k\left[e^{2bkt}\left(\psi_{OR}^{+}\psi_{OR}^{-}\cos 2\gamma - \psi_{OR}^{+}\psi_{OI}^{-}\sin 2\gamma\right)\right] \\ & + \pi k\left[e^{2bkt}\left(\psi_{OI}^{+}\psi_{OR}^{-}\sin 2\gamma + \psi_{OI}^{+}\psi_{OI}^{-}\cos 2\gamma\right)\right]\end{aligned} \tag{17}$$

where $\gamma = k a_2 t$ and π in this and subsequent equations is $\pi = 3.14159$.

The time rate of change of this kinetic energy is then obtained by differentiating (17) with respect to time with the following result:

$$\begin{aligned}\frac{\partial}{\partial t}\int_0^L \frac{V^2}{2} dx = {} & \frac{\pi k^2}{2} e^{2(b+d)kt}\left[2b\left(\psi_{OR}^{+2} + \psi_{OI}^{+2}\right) + 2d\left(\psi_{OR}^{+2} + \psi_{OI}^{+2}\right)\right] \\ & + \frac{\pi k^2}{2} e^{2(b-d)kt}\left[2b\left(\psi_{OR}^{-2} + \psi_{OI}^{-2}\right) - 2d\left(\psi_{OR}^{-2} + \psi_{OI}^{-2}\right)\right] \\ & + \frac{\pi k^2}{2} e^{2bkt}\left[4\psi_{OR}^{+}\psi_{OR}^{-}\left(b\cos 2\gamma - a_2 \sin 2\gamma\right) - 4\psi_{OR}^{+}\psi_{OI}^{-}\left(b\sin 2\gamma + a_2\cos 2\gamma\right)\right]\end{aligned} \tag{18}$$

$$+\frac{\pi k^2}{2} e^{2bkt}\left[4\psi_{oI}^{+}\psi_{oR}^{-}(b\sin 2\delta+a_2\cos 2\delta)+4\psi_{oI}^{+}\psi_{oI}^{-}(b\cos 2\delta-a_2\sin 2\delta)\right]$$

This energy change equation when evaluated at some time $t=t_i$ represents the tendency of the perturbation to gain or lose kinetic energy. This gain or loss then shows whether the wave is amplifying or dampening. Since only unstable waves are studied, a solution leading to zero corresponds to a neutral wave.

Haltiner [3] has shown that the mean energy changes over one wavelength for adiabatic conditions can be expressed as

$$\frac{\partial(\overline{KE})}{\partial t} \sim -\int \omega_2(\psi_1-\psi_3)\,dx$$

and (19)

$$\frac{\partial(\overline{PE})}{\partial t} \sim +\int \omega_2(\psi_1-\psi_3)\,dx$$

where $\overline{KE}$ and $\overline{PE}$ are the mean kinetic and potential energies. The quantity $(\psi_1 - \psi_3)$ is equal to π; so after substituting the values for C_+ and C_- into (12) and solving for ω from (4), the energy transformation functions on the right hand side (19) may be determined. The kinetic energy changes that have been examined in this paper are thus a consequence of this energy transformation from potential to kinetic energy.

III. Results

It is obvious from equation (18) that a determination of precisely how heating affects the stability characteristics of harmonic waves would be difficult. Examination of the frequency equation (10) does show that with heating present an instability term (term b) is always present, and also that heating leads to an instability term evolving from the radical part of the equation (term d). Term b always leads to a dampening effect while term d may have either dampening or amplifying characteristics. Therefore, the combined effects of these two terms will determine the energy change characteristics of the heating model. In the adiabatic model the energy change is dependent on only term d, which takes on a different value in general when heating is present.

To arrive at quantitative results for energy changes in the two situations, representative values of the different parameters involved are selected and equation (18) is solved numerically. The latitude is fixed at 45 degrees north latitude, the basic flow zonal wind remains constant at 20 m/sec, and the static stability also remains constant with $\partial\theta/\partial p$ = -6K/100mb and so $\mu^2 = 2.4 \times 10^{-16}$ cm^{-2}. The initial amplitude of the stream function and the thickness function are taken to be equal with a value of 20×10^{10} cm^2/sec which corresponds to a z-amplitude of 20km. Since both the thermal wind and phase lag are of significant importance in the energy change, four values of thermal wind (5, 10, 15, and 20 m/sec.) and 12 phase lags, 30 degrees apart, are used.

It remains only to select a representative value for the heating proportionality constant. The value for G in (7) is chosen to be

$$G = 100 \text{ ERGS } g^{-1} {}^{\circ}K^{-1} \text{ SEC}^{-1}$$

and the value for r is r=2. This is similar to values used by Döös [1] and corresponds to a total heat transfer of 63 cal per square centimeter per day to the entire column of air for every one degree temperature difference between the ground and the air at the surface. To determine a value for K, the initial perturbation thickness is translated into a temperature perturbation by use of the hypsometric equation. Since the surface air and the ground are assumed to be initially in thermal equilibrium, this temperature perturbation also equals the maximum difference between the air and the ground. A perturbation thickness of 20×10^{10} cm^2/sec which is the selected initial value translates into a temperature of 6.54K and corresponds to a heat transfer of 654 ergs g^{-1} sec^{-1}. A value for K can now be calculated from (8). The value for K at 500 millibars is then arrived at by using r=2 and turns out to be

$$K = 2.269 \times 10^{-6} \ sec^{-1}$$

Using the above values, the first items determined were the stability characteristics of the waves. As mentioned in the introduction this is the value of the complex part of the phase velocity multiplied by k, where $k = \frac{2\pi}{L}$. The results are summarized in figures 1 through 4. Figure 1 is for no heating and a thermal wind of 10m/sec. Figure 2 is again for no heating and now the thermal wind is 20m/sec. Figures 3 and 4 are the corresponding situations for the heating model.

To gain sufficient resolution between the values for each model at interested wave lengths the graphs were scaled such that the values at short wave lengths often exceeded the bounds of the graph. Where the bounds have been exceeded such that the graph plot is a straight line parallel to the x-axis, the actual values at these wave lengths are greater than those depicted on the graph. In cases where the values

are significant to the problem, special mention will be made of the values. It should also be mentioned that the data points are obtained at intervals of 1000 kilometers and between data points the computer simply makes straight line extrapolations.

Examination of figures 1 through 4 reveals the following items. Heating leads to a smaller amplification factor than no heating and a larger dampening factor. The values at 5000km, which are off the graph for a thermal wind of 20m/sec, also show that the no heating leads to a larger amplification and less dampening. The effect of thermal wind is readily seen to increase the instability characteristics even though the value of the b term in the heating model is independent of thermal wind. Also it is important to note that in the adiabatic case the instability is confined to a finite wave length interval which depends on the thermal wind and static stability. In contrast the presence of heating leads to an instability factor at all wave lengths, however the amplification approaches zero with increasing wave length. These results are in agreement with similar computations made by Haltiner.

Figure 5 depicts the relationship between the speed of wave propagation in the two models. Ignoring the value at wave length zero for the heating case, heating leads to a decrease of phase velocity of the wave at all wave lengths. This difference is, however, quite small at 4-6000 km. This also is in general agreement with results obtained by Haltiner.

Computations of the time rate of change of kinetic energy show some interesting features. First of all, if the energy change tendency at time t=0 is evaluated for several different values of phase lag, the maximum positive effect occurs at a phase lag of 90 degrees and the maximum negative change is at a phase lag of 270 degrees. The pattern is sym-

metrical about these two points for both models with the exception that for the adiabatic case the values are identically zero at phase lags of 0 and 180 degrees while in the case of heating they are small but not identically zero. A plot of the kinetic energy tendency at time t=0 for several different phase angles is shown in figures 6 and 7. As the time was increased from zero the phase lag of maximum positive and maximum negative energy change tended to increase, so that by 12 hours several situations (wave length and thermal wind values) showed a maximum positive value at 120 degrees of lag and maximum negative at 300 degrees. Also, in all situations the magnitude of the positive rate of change was greater than that of the negative change. Again the graphs indicate that the presence of heating leads to kinetic energy changes at all wave lengths while its absence restricts the energy change to a finite interval. This phenomena remains true throughout all situations investigated.

Another point that should be brought out at this time is that in the derivation of equation (18), both wave components of (11) and (12) were used. The energy change equation, therefore, represents the combination of the damped and amplifying components. Hence, the values of wave length, thermal wind, and phase lag which result in positive energy change represent situations when amplification will take place and vice-versa.

When computations for the kinetic energy tendency for the heating and non-heating case were compared at t=0, the values were identical for corresponding situations (with the exception of phase lag 0 and 180 degrees which was previously mentioned). This seemed quite surprising at first since equation (17) for the diabatic and adiabatic cases is quite different. Upon further reflection it is apparent that since

heating has had no time to exert an influence, the values would be the same. Figures 8 and 9 show the tendency at t=0 for several thermal winds. On the curves for a thermal wind of 20m/sec, the values at wave lengths 5000, 6000 and 7000 kilometers can be seen to be equal in both cases.

Next the tendency is evaluated at t=6 hours. In both cases the values increased in magnitude but now an examination of figures 10 and 11 show that the values for the heating case are smaller than those for no heating. When time is allowed to increase further, the difference in tendency values for the two cases becomes larger. Figures 12 through 15 are for t=12 and t=24 hours and show how the difference increases. By 24 hours the values at 5000 kilometers are off the graphs but the raw data still gives a higher value for the adiabatic situation.

Values of energy change were actually tabulated out to 33 hours at a phase lag of 90 degrees. In the adiabatic model the tendency value increased out to 33 hours in all cases. When heating was present the value also increased with time for medium wave lengths with the exception that after 30 hours for a wave length of 5000 kilometers and a thermal wind of 5 and 10m/sec, the value started to decrease. This suggests that not only does the tendency increase at a slower rate when heating is introduced, but also that it reaches its maximum value sooner. At the longer wave lengths the value of the tendency decreases with time such that by 24 hours kinetic energy is actually being lost by the waves with wave lengths larger than 10,000 to 11,000 kilometers. It was mentioned earlier that the phase lag of maximum energy change increases with time also. This change of phase lag does not, however, alter the results that heating leads to smaller values of tendency as the time increases.

So far we have examined primarily the spectrum of medium length waves

to determine the effects of heat transfer on the energy changes. At the long wave lengths the addition of heat transfer to the model leads to only a small rate of energy change. This, however, is significant since in the adiabatic case the energy change is identically zero at long wave lengths. That heating insured the presence of an instability term at all wave lengths was observed in equation (10) and again in the results of figures 3 and 4. These did not, however, indicate whether the effect was to amplify or dampen the wave. Examination of the kinetic energy tendency due to heating clearly shows that the effect depends on the time and the phase lag. For example, the series of graphs on energy change show that out to at least 24 hours, if the phase lag is 90 degrees, the heating leads to an amplifying wave at long wave lengths. Beyond 30 hours the heating leads to a dampening effect.

The energy tendency at the very short wave lengths was not examined comprehensively and no explanation is offered for the very large values of energy change observed. It can be seen from the graphs, however, that besides displaying very large values, the kinetic energy tendency may change abruptly from a negative to a positive value or vice versa as the time or wave length varies.

ACKNOWLEDGEMENTS

The writer wishes to express his appreciation to Professor George J. Haltiner for his help and counsel in bringing this work to a completion.

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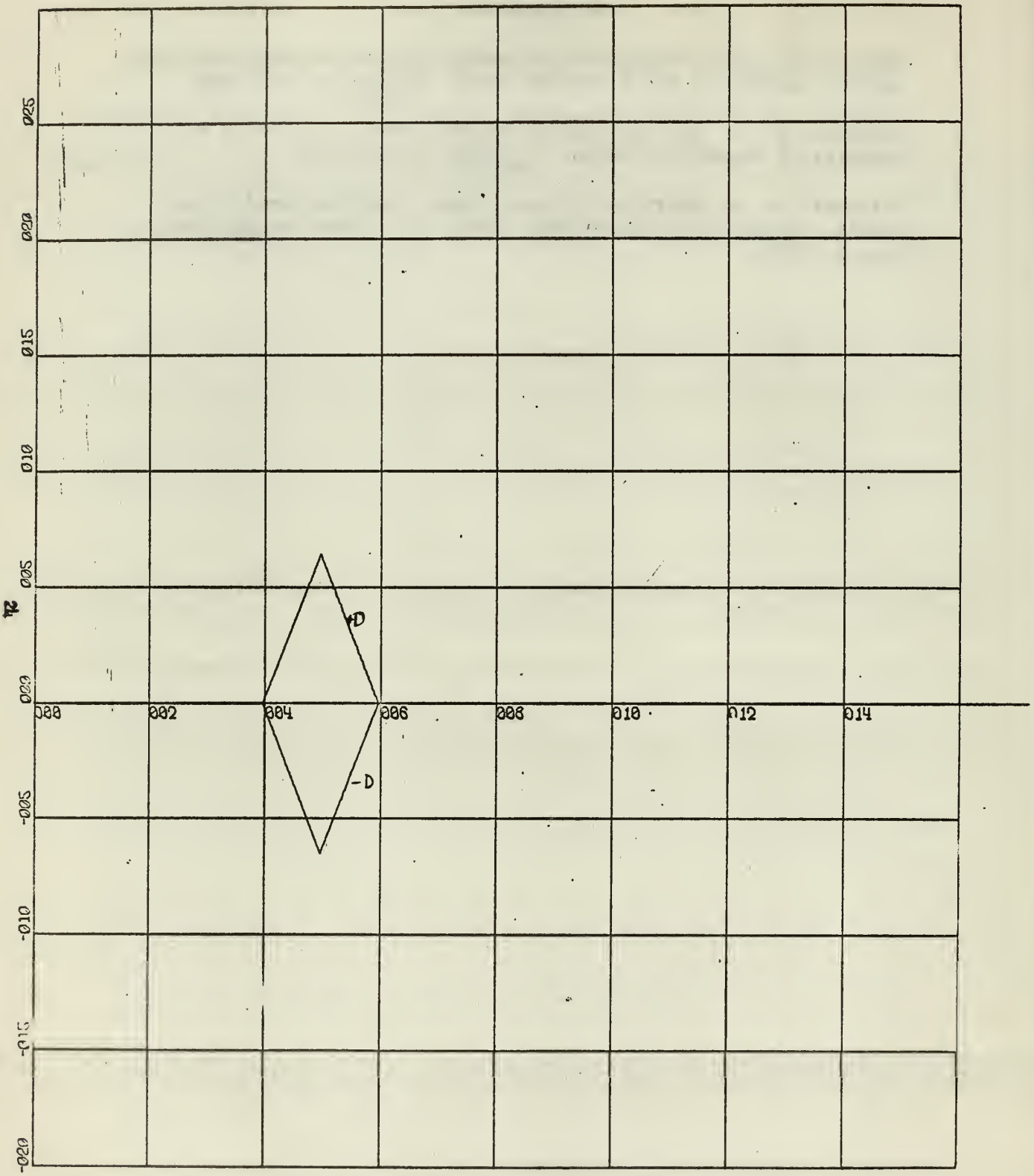
3. Haltiner, G. J. Energies of some simple numerical prediction models, unpublished departmental notes, U.S. Naval Postgraduate School, 1966.



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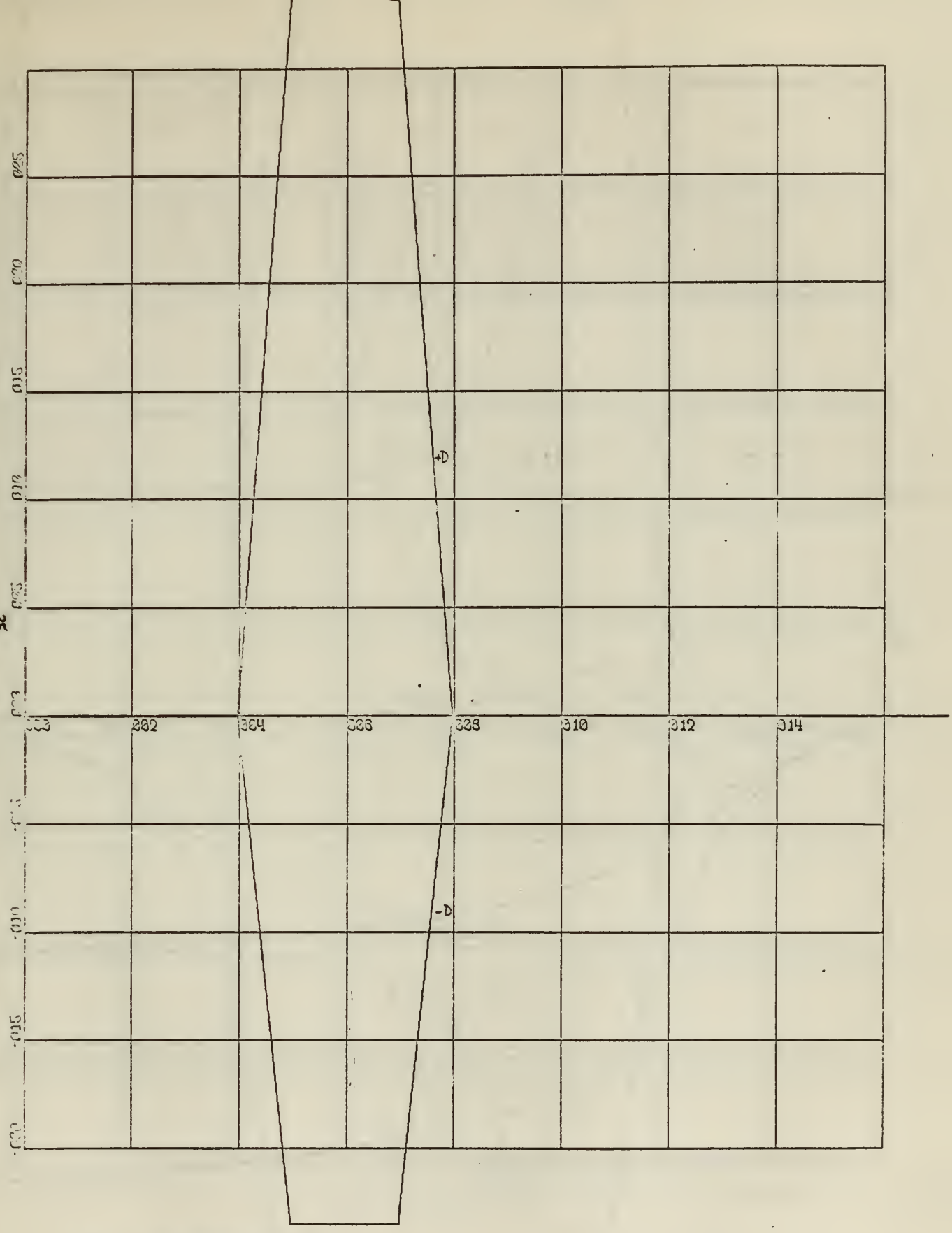
Y-SCALE = 5.00E-07 UNITS/INCH.

AMPLIFYING (+D) OR DAMPENING (-D) FACTOR AS A FUNCTION OF WAVELENGTH THERMAL WIND 10M/S NH

Figure 1



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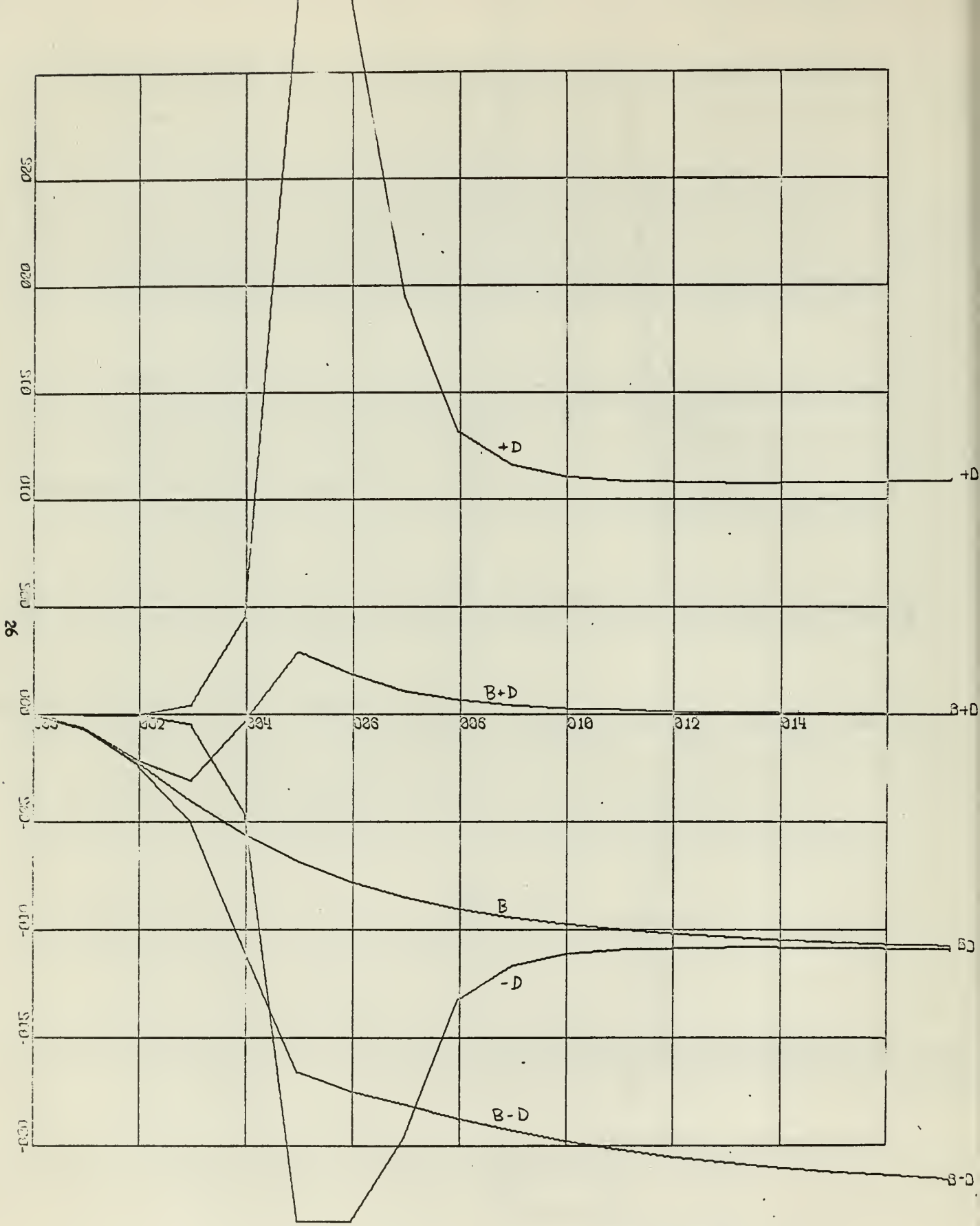
Y-SCALE = 5.00E-07 UNITS/INCH.

AMPLIFYING (+D) OR DAMPENING (-D) FACTOR AS A FUNCTION OF WAVELENGTH THERMAL WIND 20M/S NH

Figure 2



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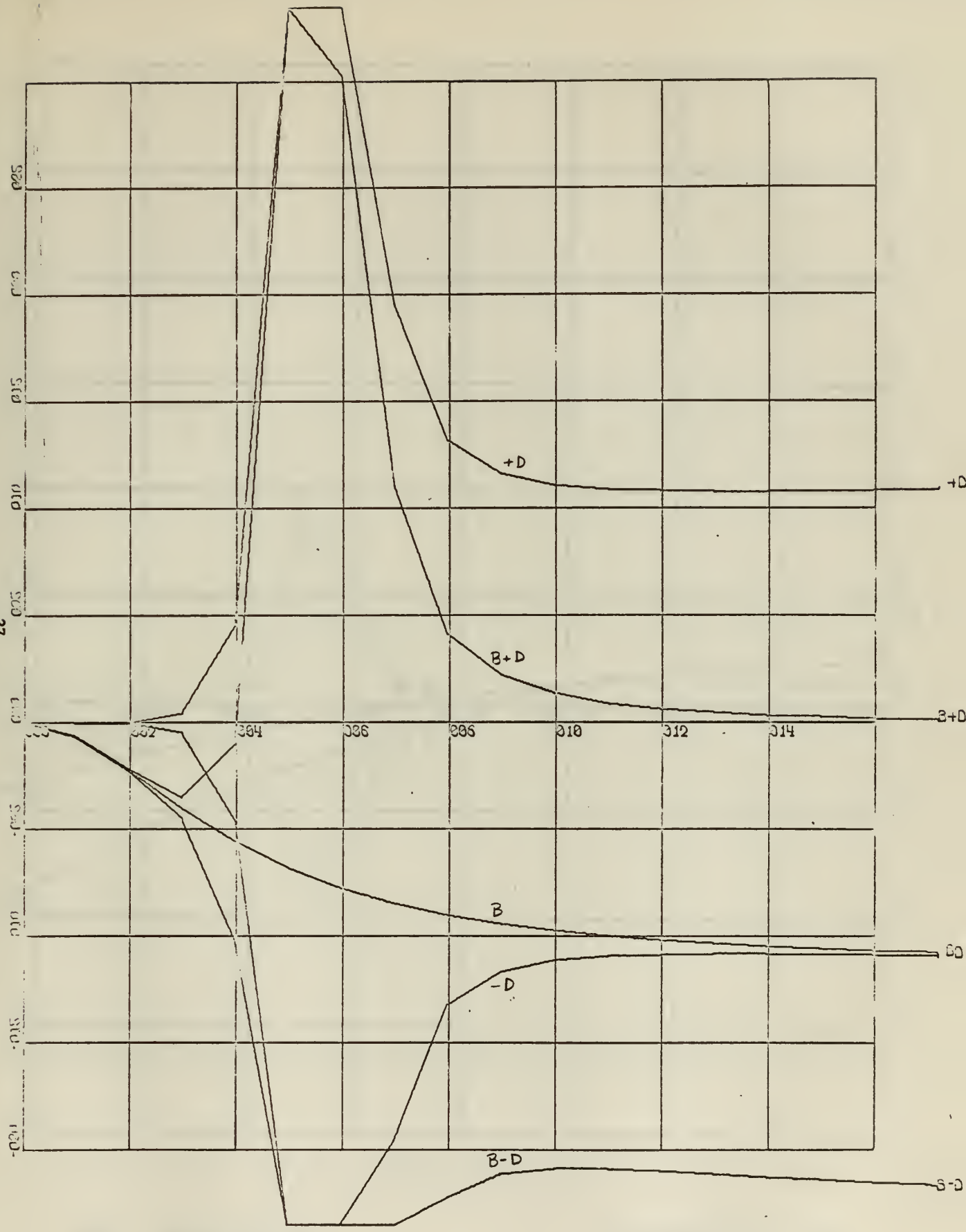
Y-SCALE = 5.00E-07 UNITS/INCH.

AMPLIFYING OR DAMPENING CHARACTERISTIC B+D, B-D, B, +D, -D FOR THERMAL WIIND 10M/S HEATING

Figure 3



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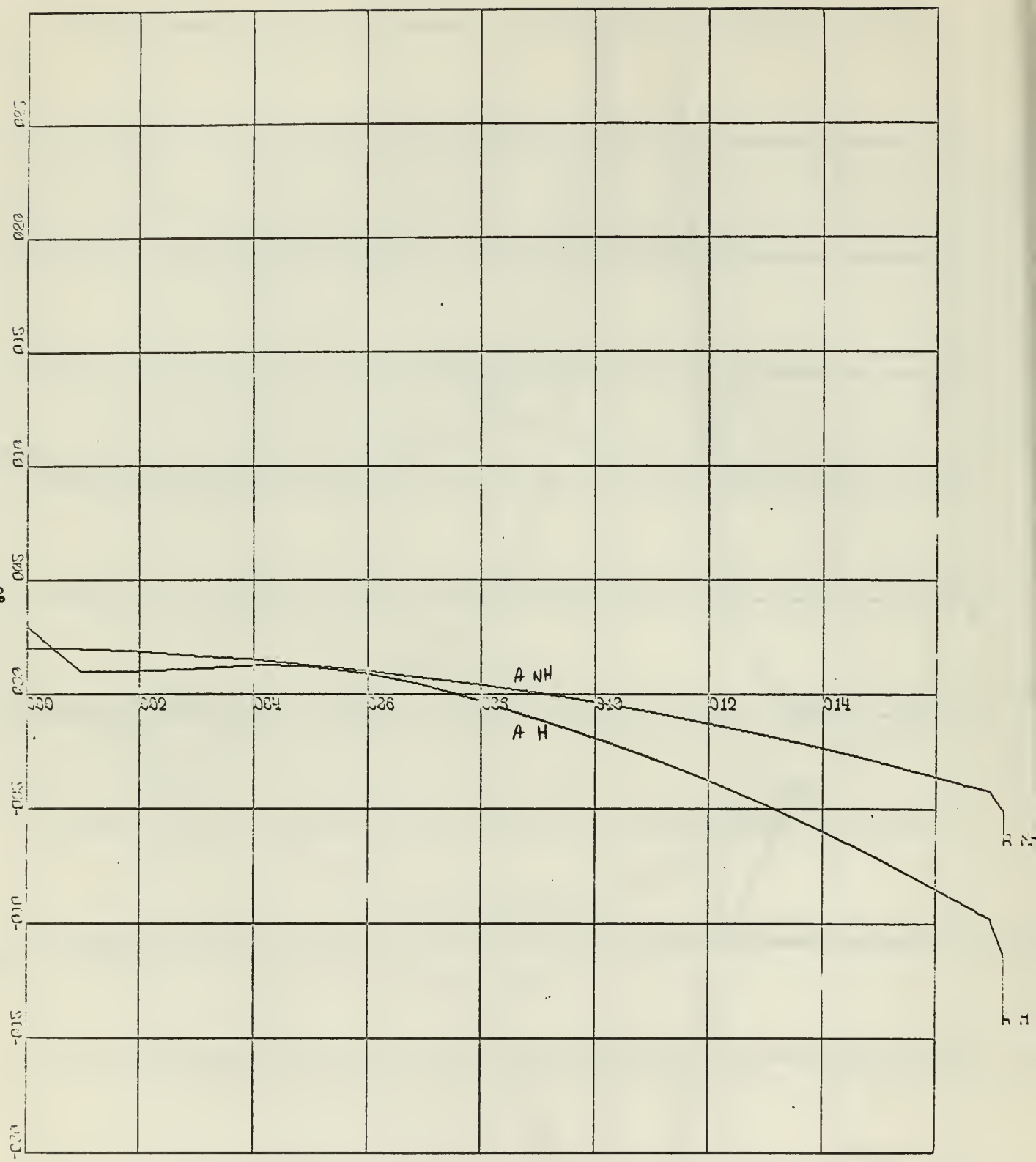
Y-SCALE = 5.00E-07 UNITS/INCH.

AMPLIFYING OR DAMPENING CHARACTERISTIC B+D, B-D, B, +D, -D FOR THERMAL WIND 20M/S HEATING

Figure 4



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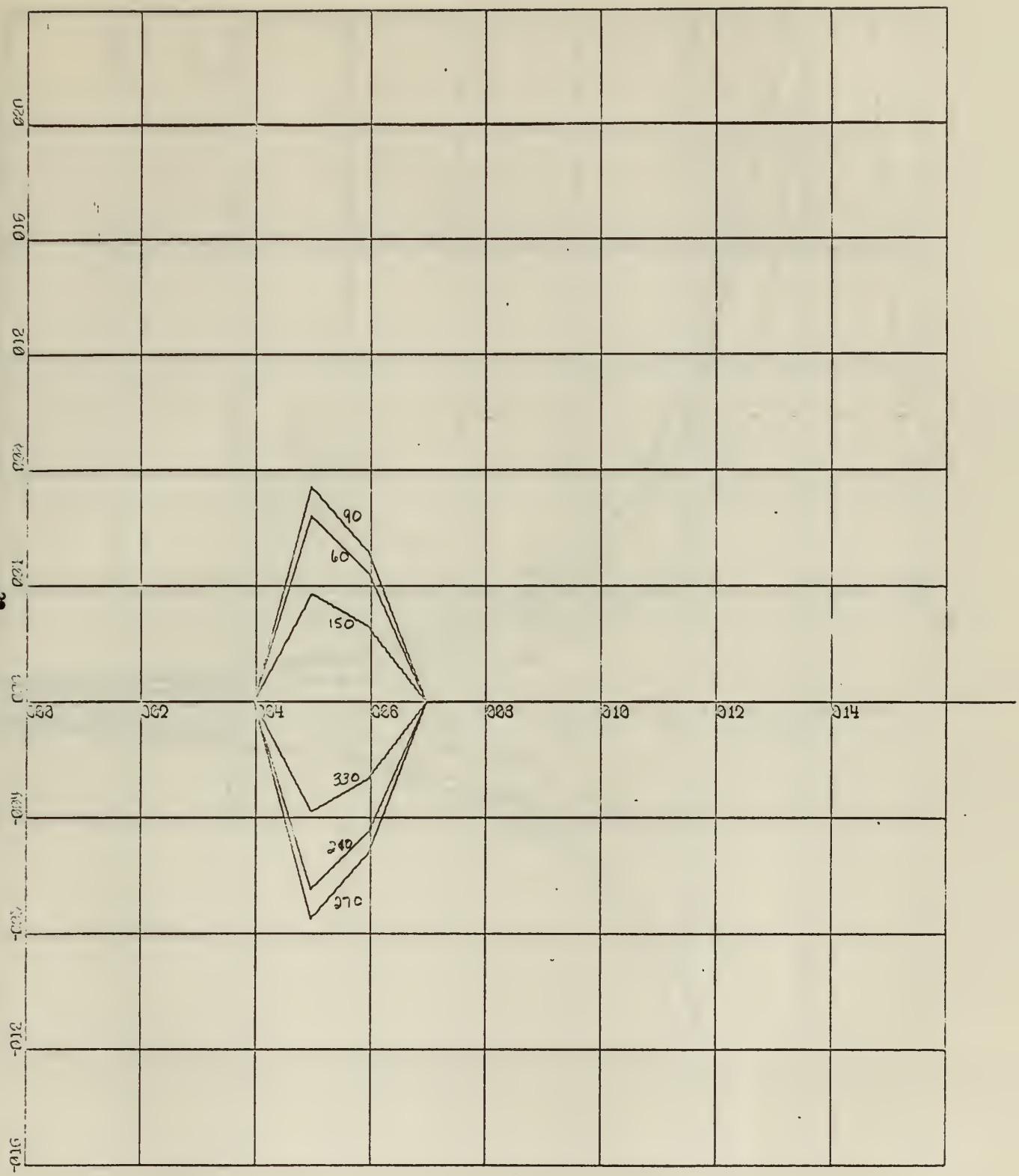
WAVE SPEED HEATING (A H) VS NOHEATING (A NH)
THERMAL WIND 20 M/S

Figure 5



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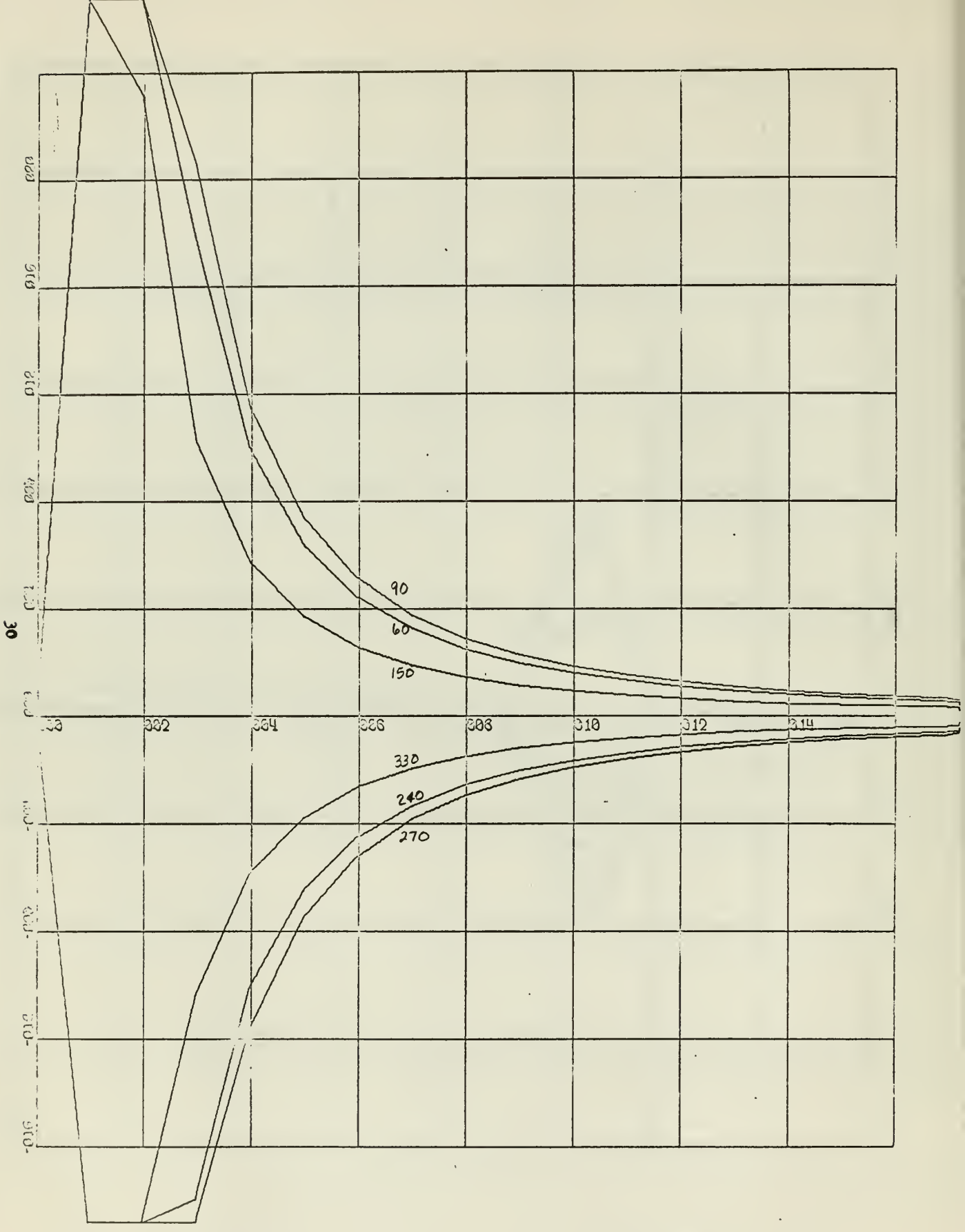
Y-SCALE = 4.00E+09 UNITS/INCH.

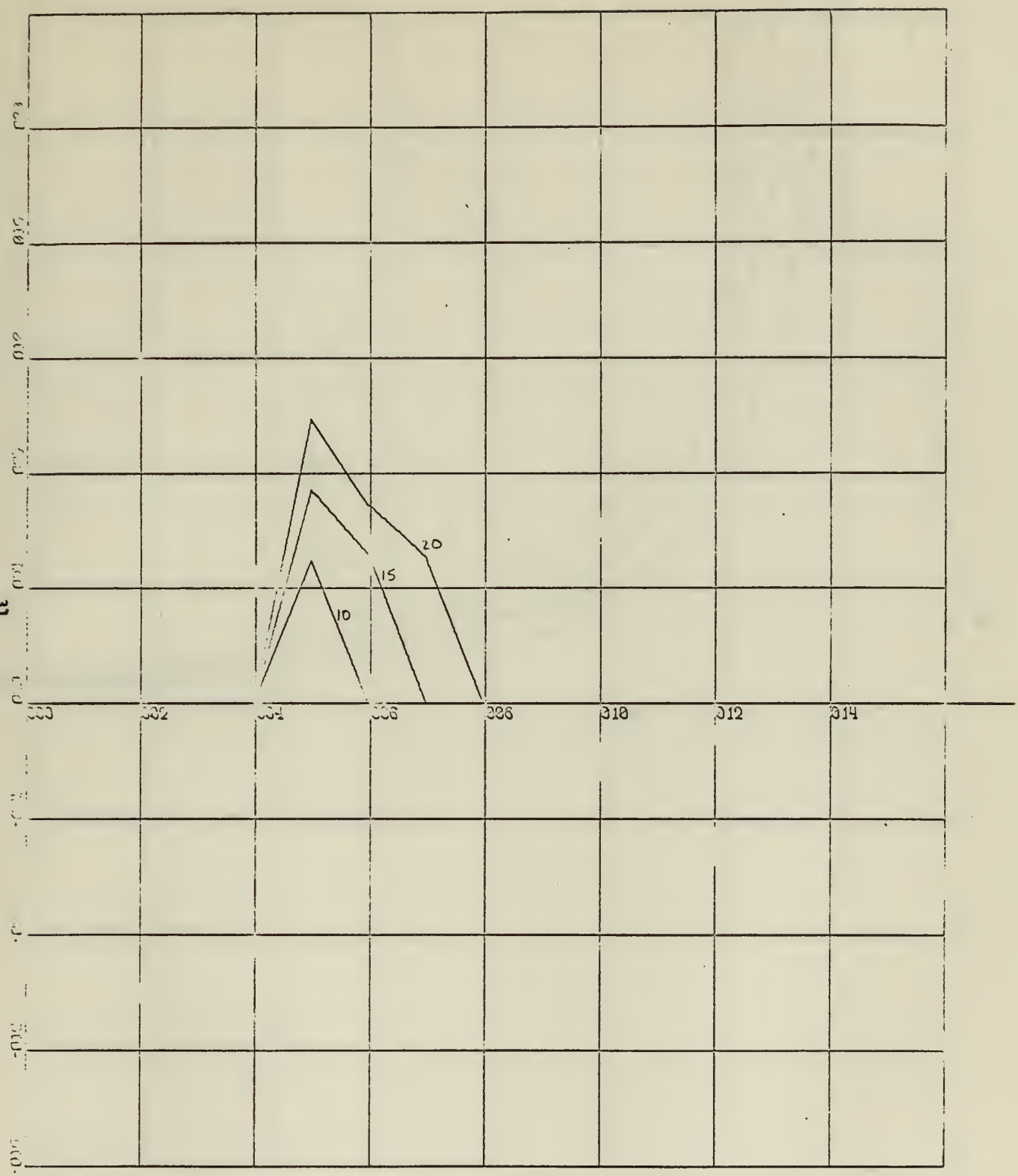
KE TENDENCY (ERGS/S) FOR PHASE LAG 60, 90, 150, 240, 270, AND 330 DEGREES THERMAL WIND=15M/S NH

Figure 6



KE TENDENCY (ERGS/S) FOR PHASE LAG 60, 90, 150, 240, 270, AND 330 DEGREES THERMAL WIND=15M/S H

Figure 7



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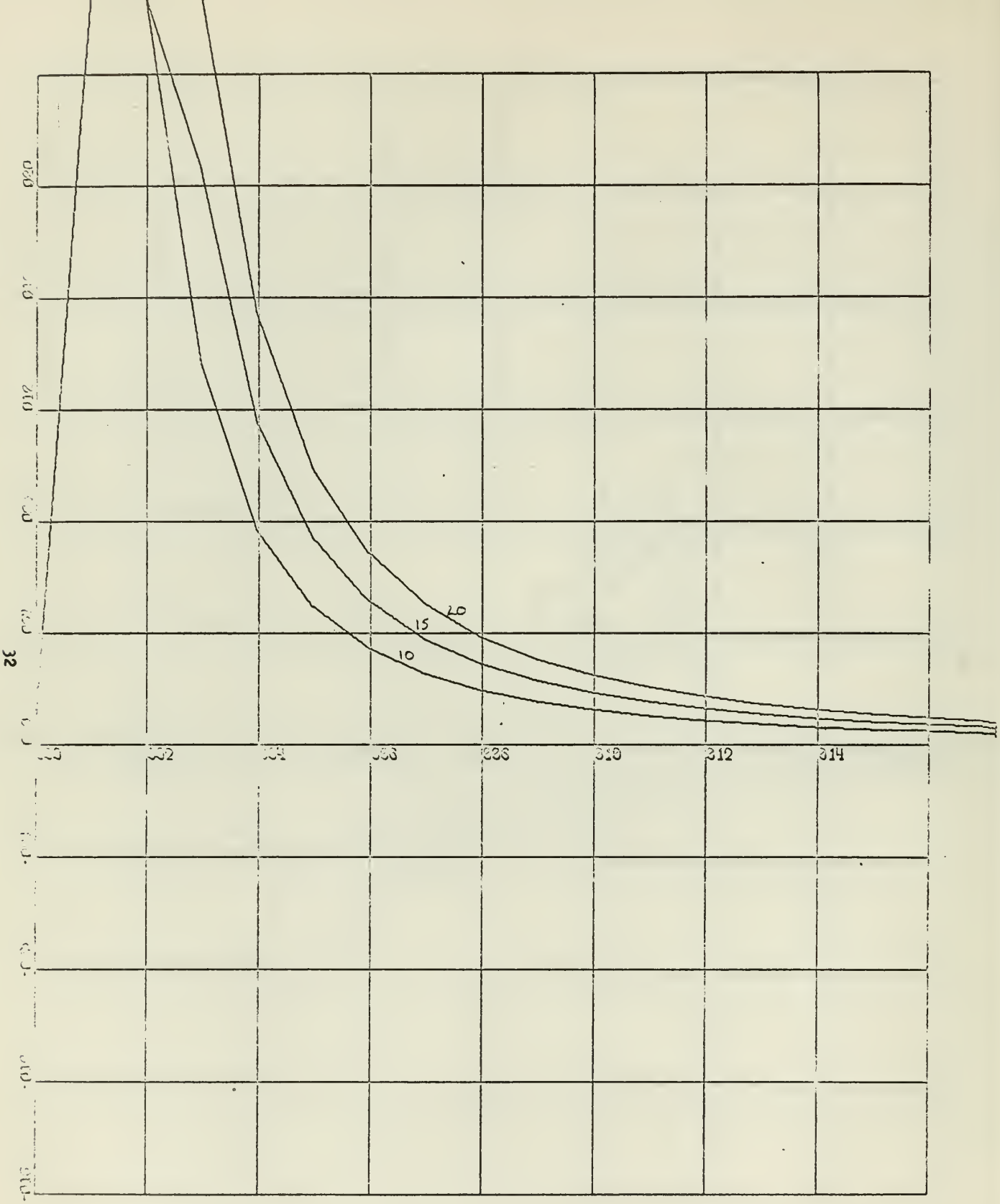
Y-SCALE = 4.00E+09 UNITS/INCH.

KE TENDENCY (ERGS/S) FOR THERMAL WINDS 10, 15, AN
D 20M/S AT 0 HRS AND PHASE ANGLE 90 DEG NH

Figure 8



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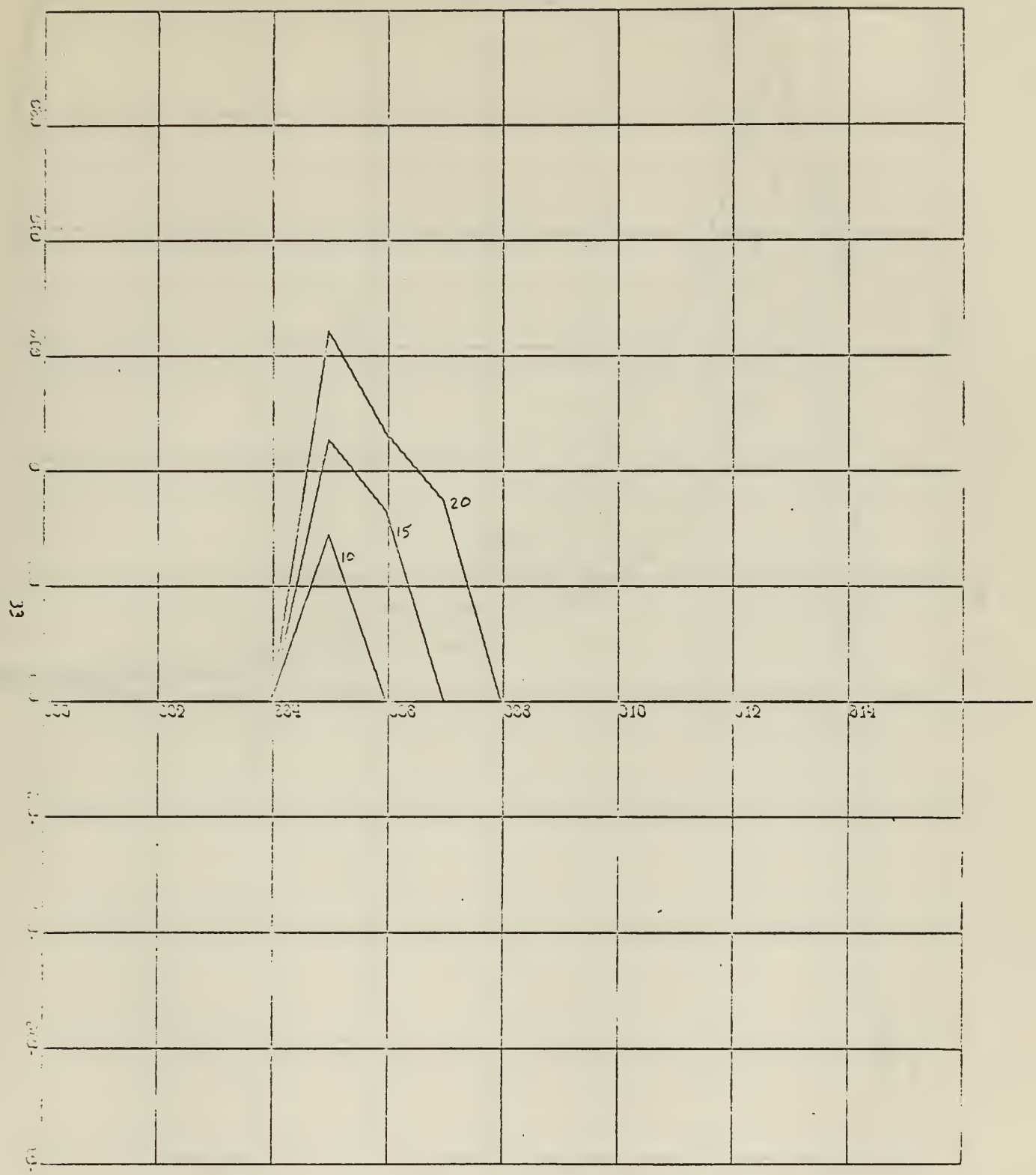
Y-SCALE = 4.00E+00 UNITS/INCH.

KE TENDENCY (ERGS/S) FOR THERMAL WINDS 10, 15, AN
D 20M/S AT 0 HRS AND PHASE ANGLE 90 DEG H

Figure 9



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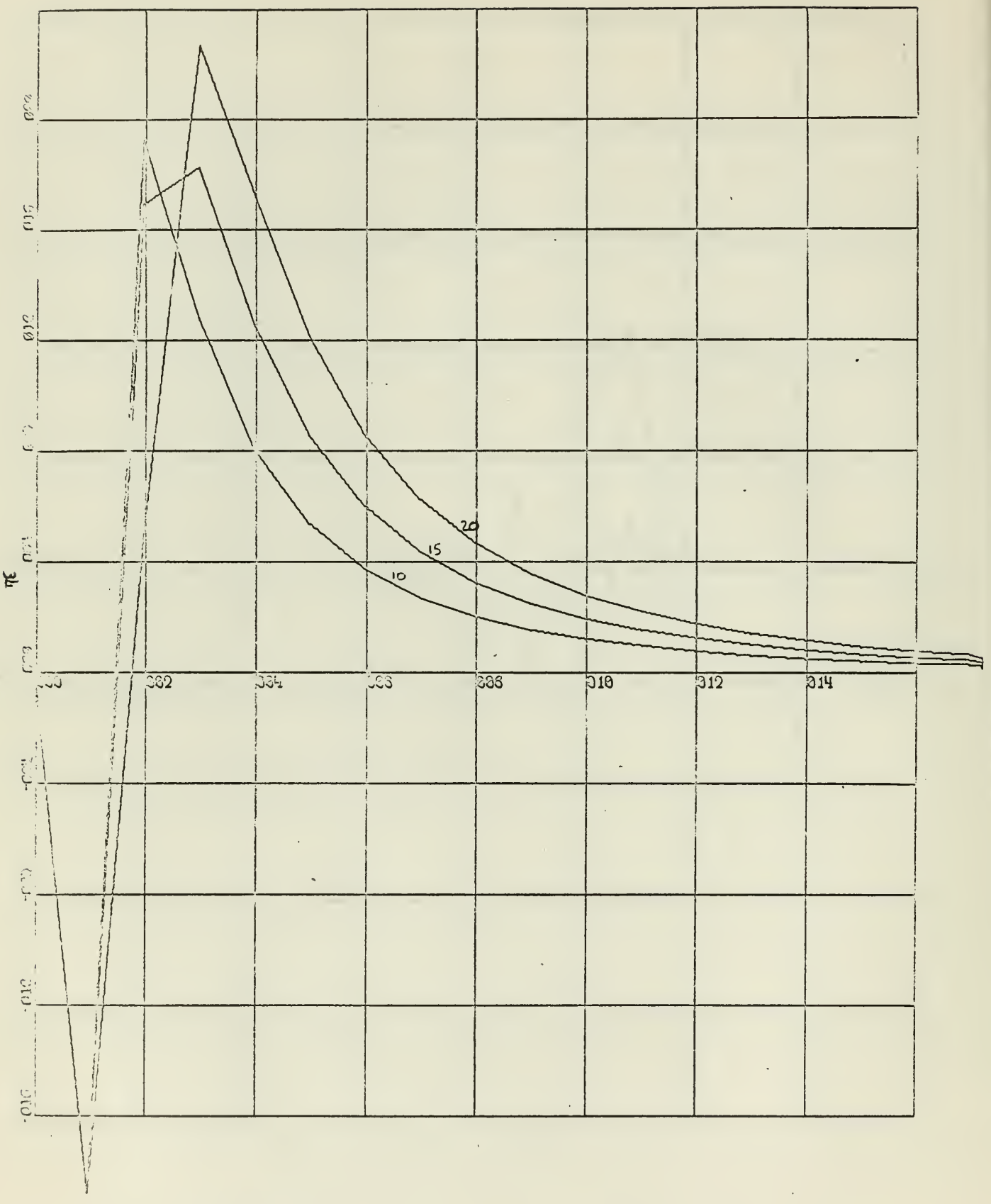
Y-SCALE = 4.00E+00 UNITS/INCH.

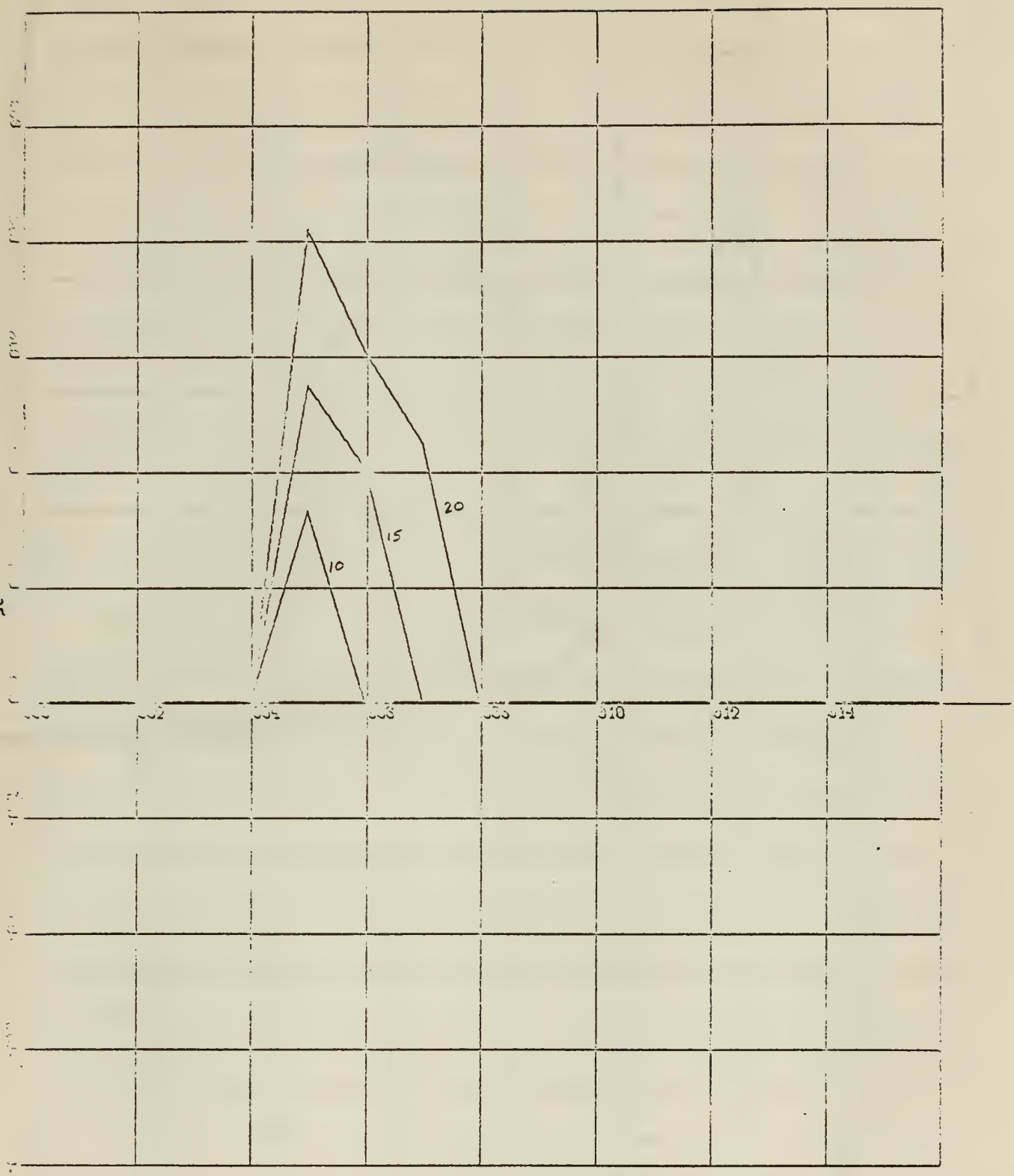
KE TENDENCY (ERGS/S) FOR THERMAL WINDS 10, 15, AN
D 20M/S AT 6 HRS AND PHASE ANGLE 90 DEG NH

Figure 10



KE TENDENCY (ERGS/S) FOR THERMAL WINDS 10, 15, AN
D 20M/S AT 6 HRS AND PHASE ANGLE 90 DEG H

Figure 11



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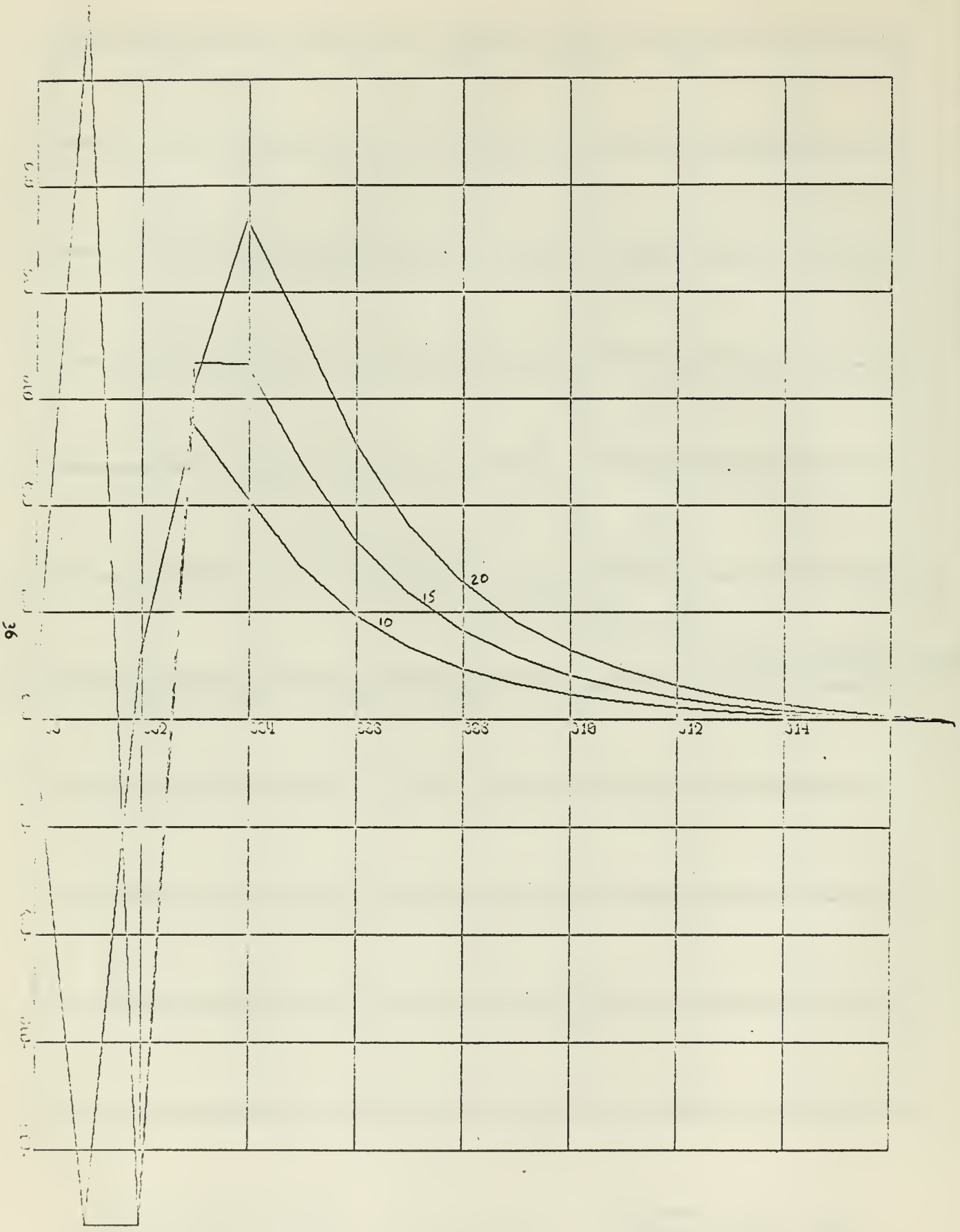
Y-SCALE = 4.00E+09 UNITS/INCH.

KE TENDENCY (ERGS/S) FOR THERMAL WINDS 10, 15, AN
D 20M/S AT 12 HRS AND PHASE ANGLE 90 DEG NH

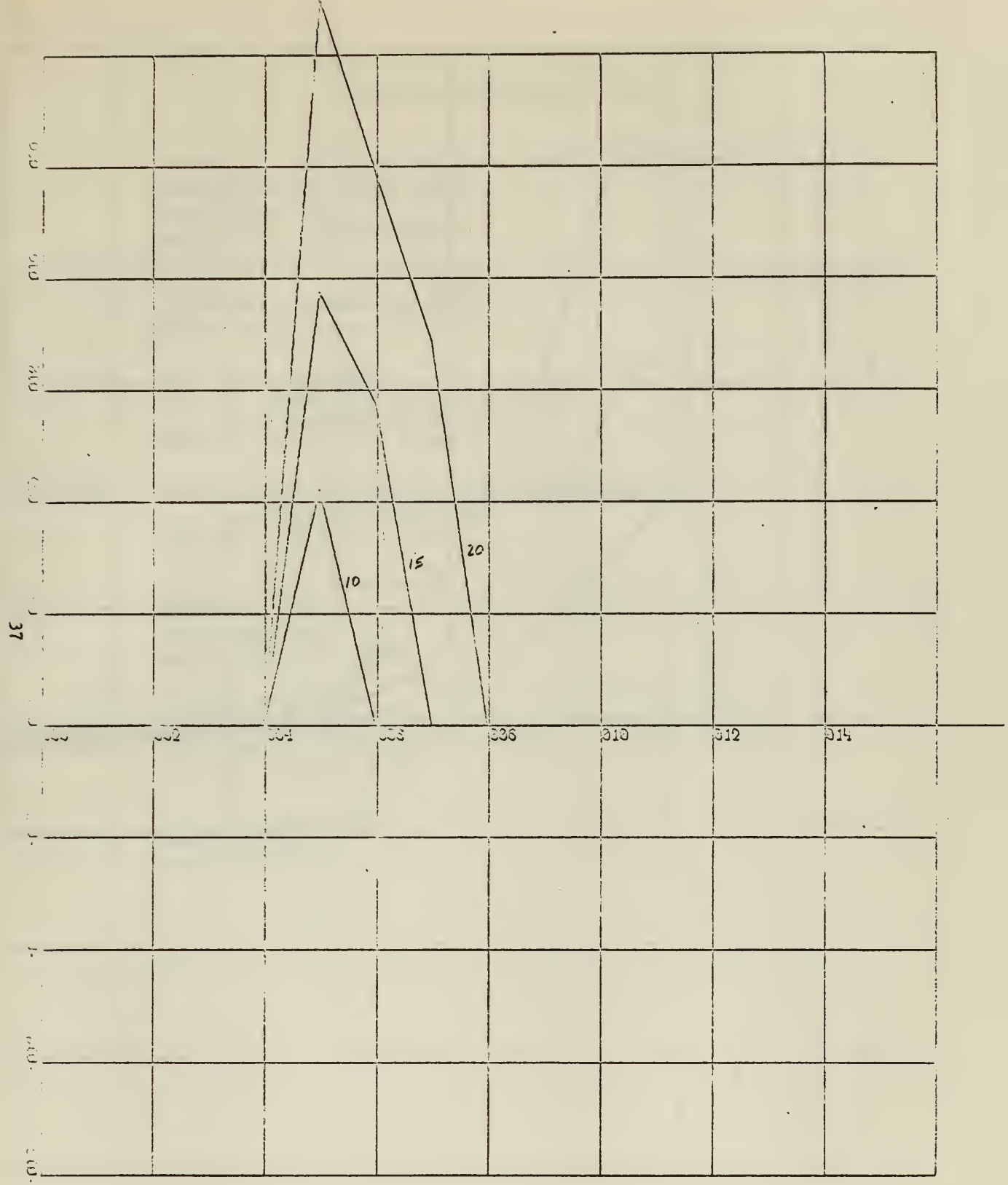
Figure 12



X-SCALE = 2.82E+03 UNITS/INCH.
Y-SCALE = 4.82E+03 UNITS/INCH.

KE TENDENCY (ERGS/S) FOR THERMAL WINDS 10, 15, AN
D 20M/S AT 12 HRS AND PHASE ANGLE 90 DEG H

Figure 13



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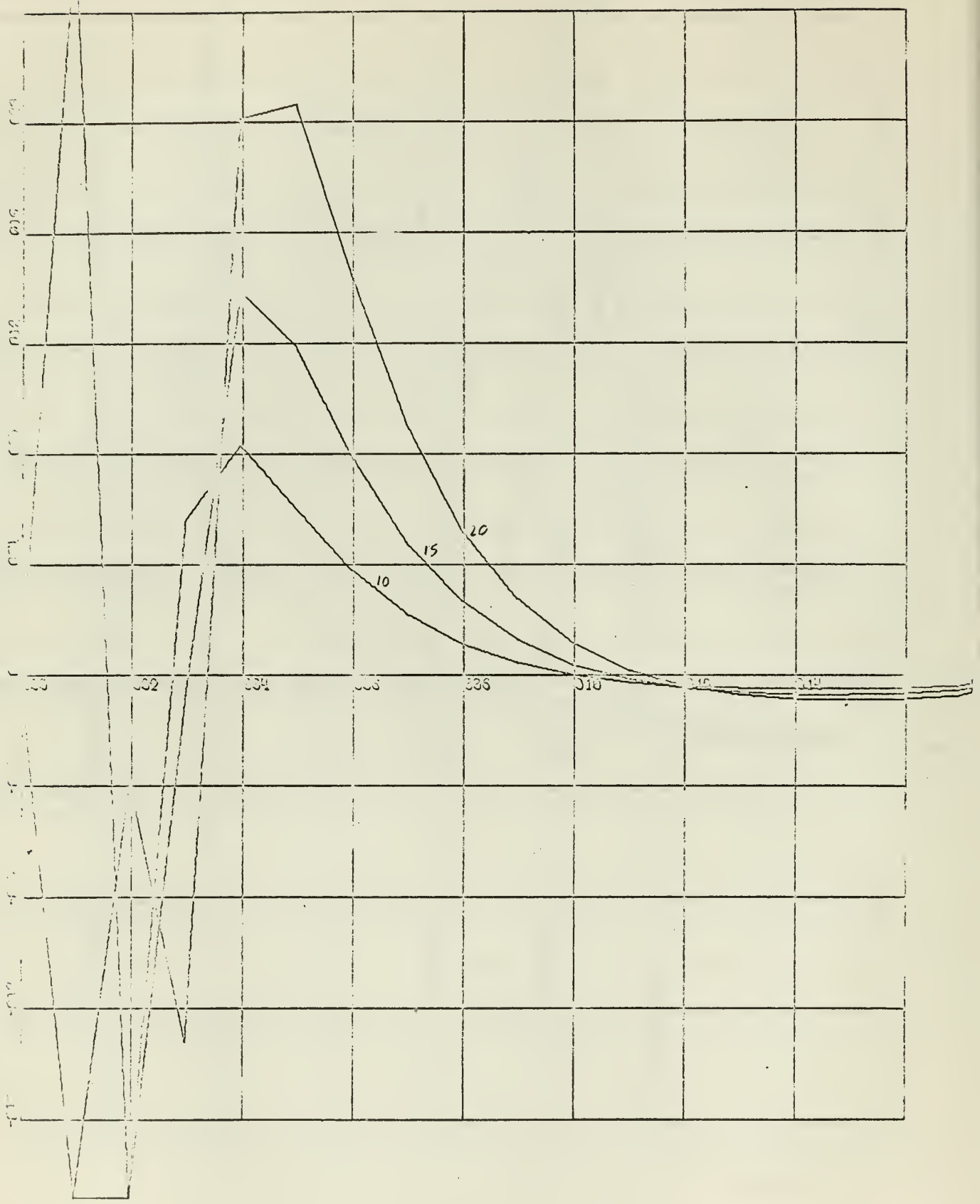
Y-SCALE = 4.03E+00 UNITS/INCH.

KE TENDENCY (ERGS/S) FOR THERMAL WINDS 10, 15, AN
D 20M/S AT 24 HRS AND PHASE ANGLE 90 DEG NH

Figure 14



X-SCALE = 2.88E+08 UNITS/INCH
Y-SCALE = 4.00E+08 UNITS/INCH

KE TENDENCY (ERGS/S) FOR THERMAL WINDS 10, 15, AN
D 20M/S AT 24 HRS AND PHASE ANGLE 90 DEG H

Figure 15

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2a. REPORT SECURITY CLASSIFICATION
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2b. GROUP

3. REPORT TITLE

ENERGY CHANGES IN BAROCLINIC MODELS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Thesis, M.S., October 1966

5. AUTHOR(S) *(Last name, first name, initial)*
HUEHN, Kelvin Wayne

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
October 1966	41	3

8a. CONTRACT OR GRANT NO.

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9a. ORIGINATOR'S REPORT NUMBER(S)

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The effects of heat transfer on the kinetic energy tendencies of a perturbation superimposed on a basic zonal current are investigated. The rate at which kinetic energy is gained in both the adiabatic and diabatic models increases with time for medium wave lengths, but the rate of increase is less for the diabatic case. At the longer wave lengths the energy change for the diabatic model shows a decrease with time becoming negative toward the end of the time interval investigated. The adiabatic model, in contrast, has zero energy tendency at long wave lengths. The rate of kinetic energy change increases as larger values of thermal wind are selected, and for the adiabatic model the wave length range for which energy changes occur also expands with increasing thermal wind. The energy gain initially is a maximum when the thermal wave lags the pressure wave by a phase angle of 90 degrees and the maximum tendency to lose energy occurs when the lag is 270 degrees.

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